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Editors

International Centre for Mechanical Sciences

Instabilities of Flows: with and without Heat Transfer and Chemical Reaction

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INTERNATIONAL CENTRE FOR MECHANICAL SCIENCES

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INSTABILITIES OF FLOWS: WITH AND WITHOUT HEAT TRANSFER AND CHEMICAL REACTION

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Prefaces

This monograph is an outcome of the summer school organized on behalf of CISM in June 2008, for the advanced course titled "Instabilities of Flows with and without Heat Transfer and Chemical Reactions". The course was conducted over five days of a working week in the beautiful and peaceful surrounding of Udine with an *ashram*-like atmosphere and the course was held quite intensively, with active participation of the lecturers and all the participants, including one of the rectors of the institute.

The scope of the course was vast and the lectures turned out to be reflections of the expertise of the lecturers. The material covered in the course ranged from classical hydrodynamic instability (although taught from asymptotic theory perspective) to newer areas of interests and finally to the complex field of flow instability involving heat transfer and chemical reactions. While significant progresses have been made over the last century and half in understanding the field of instability, the motivation still remains that is given by the aphorism: *the flow that occurs in nature must not only follow the equations of fluid dynamics, but also be stable* (Landau & Lifshitz, 1959).

Currently there are more than one monographs and textbooks available that deals with the basic subject of instability and a question arises naturally, if there is a need for another monograph. Before proposing the course, I discussed this aspect with Prof. Schneider who actually encouraged us to propose what we ended up doing: To teach a course on the subject of our expertise that will have minimal overlap with available materials, at the same time to remain relevant to the needs of the participants. The enthusiasm of the participants in the summer school convinced us that the goal was met by and large. That also encourages us to go ahead and put those materials in one place in this monograph those happen to be the research interests of the contributing authors. In presenting this monograph, as the editor, I realise that if a scholar has nothing new to say, then he should keep quiet. I fully realise that some of the materials presented in the manuscript is alternative ways for the quest for truth and if they provoke the readers to think differently, then the purpose for the monograph will be more than met. The monograph essentially is in two parts, as contributed by the two authors. The first part is a synthesis of classical material mixed with emerging areas of research, but dealt with in a completely different manner than that is found in other books and monographs. The second part is also a mixture of classical physical ideas as applied to a very complex engineering problem, addressed by computational means.

Instability and transition have a track of torturous development, immediately following the successful explanation of inviscid mechanisms by Kelvin, Helmholtz and Rayleigh- who assumed that viscous action can only be stabilizing. The subject of instability suffered from this misconception for quite a long time. Notable researchers who were victims of indifference by proposing alternate ideas were many, including Heisenberg. His fate was followed by the researchers belonging to the German school led by Prandtl. When this was eventually circumvented in 1940's with viscous instability theory accepted as an established area of research, it was found to be moderately good enough for many more decades to come. But then *a thing moderately good is not so good as it ought to be (Tom Paine).*

The overbearing successes of linear instability theory impeded development in other important areas of (i) receptivity and (ii) many other mechanisms of transition, e.g. bypass transition and spatio-temporal growth of disturbances seen in flows where linear theories apply. In chapter 2, we have dealt with a unified description of instability and receptivity- which has not been dealt systematically before. This should be considered a first for this monograph.

Technological advances in aerospace industry were achieved by the linear instability theory, whose sole criterion of existence of Tollmien-Schlichting waves as disturbances have dominated the post-world war II research in this area. Any routes of transition other than that by TS waves were christened as bypass transition by Morkovin in late 1950. It meant many things to many researchers and the resultant lack of focus seems to pervade the field even today. We have tried to state the case of one mechanism of bypass transition systematically in chapter 3 from the first principle. In developing this area, we have also revived the energy-based stability and receptivity concept from a completely new interpretation of mechanical energy, and not relying on kinetic energy alone. The classical energy-based stability theory based on only kinetic energy is known to be deficient! This has been rectified in recent times and the appearance of a new energy based theory is given in chapter 3 and 4.

The other canonical flow geometry considered in the first part consists of bluff-body flow instability dealt in chapter 5. This introduces the flow past a cylinder that actually suffers linear temporal instability moderated by nonlinear stabilization. This flow is different from that is discussed primarily in chapters 2 to 4, where the linear instability is via spatial growth. Also, for such flows nonlinearity leads to further destabilization, whereas for the flow past a cylinder, the nonlinearity stabilizes the linear instability and takes the flow to another equilibrium flow. In chapter 6, the effects of heat transfer via the restrictive condition of Boussinesq approximation for the canonical flow past flat plates is studied. This problem has been solved by the robust compound matrix method (CMM), revealing newer insight. Advancing CMM as a method for solving stiff differential equations arising in instability problem is also a significant new addition to the subject, as given in this monograph.

In a complete departure to the first six chapters, the last four chapters show the practical method of studying flows involving combustion. Such flows and their instability at the present time are solved numerically and this approach is followed in these chapters. Nonetheless, the author pays particular attention to subtle issues of wave propagation in reacting flows and their special simulation techniques. The author particularly emphasizes on studying multiphase flows in combustion chambers by newer innovative numerical techniques. Not content with just simulating such flows numerically by LES, the author also investigates numerical instabilities in such simulation by often neglected round-off error. This is a significant addition to the subject area to deal with complex flows numerically and should help practising scientists and engineers involved in research on the topic of combustion and its instability.

TK Sengupta wants to acknowledge the help provided by V.K. Suman, Neelu Singh, Y. Bhumkar, S. Bhaumik and Kamal K. Mishra for help with preparation of the manuscript.

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The theory of instability of fluid flows is one of the oldest branches of fluid mechanics. Results that are considered as fundamental even at present days were already obtained in the 19th century, and the names of eminent scientists like Helmholtz, Kelvin, Rayleigh and Reynolds are associated with those early results. Remarkably, instabilities of fluid flows have not lost their appeal to the fluid mechanics community up to this day. Those colleagues who are fond of great challenges might consider the subject so fascinating because it is so difficult, but there are other reasons that may be of relevance, e.g. the importance of flow instabilities in many applications, including many branches of engineering. After all, steady-state solutions of the equations of motion, whether they might have been obtained analytically as so-called exact solutions, or numerically by means of CFD, will be observed in the real world only if they are stable.

Those, among others, were the reasons why the Scientific Council of CISM unanimously accepted Professor Sengupta's proposal for an Advanced Course entitled "Instabilities of Flows with and without Heat Transfer and Chemical Reactions". For the favourable decision of the Scientific Council it was also essential that Professor Sengupta himself, an authority in the field, was available for organizing the course, which included the important task of proposing eminent, internationally recognized scientist as lecturers.

The course organized, and directed, by Professor Sengupta was a great success, not only in terms of the number of participants, but also from a scientific point of view. I myself had the privilege of representing CISM at the course. I attended presentations of all lecturers, and it was a pleasure to see the participants—of various levels of professional experience—follow the lectures with great interest, themselves also making useful contributions in the discussions. I should like to add that, like the participants, I also learned a lot from the lectures that were both instructive and stimulating.

The Rectors Committee, which comprises the rectors, the former rectors, the secretary general and the vice secretary general of CISM, has always been in favour of publishing the lecture notes of each course in the Springer book series "CISM Courses and Lectures", with the course organizer(s) acting as editor(s). Thus the lecturers, including the organizer, are usually asked to submit manuscripts that are suitable for publication in the series, accounting for the results of the discussions during the course, if appropriate. However, not always agree all lecturers to have their course notes published. The present case is a particular one. Despite great efforts by Professor Sengupta as the editor, only two, out of a total of five, lecturers - including the editor himself - were willing, and able, to submit manuscripts in due time. The result, however, is nothing less than an excellent contribution to the CISM book series. Professor Sengupta's article has, in fact, developed into a monograph on the subject of his lectures, and Dr. Poinsot has also nicely refined the interesting material he had presented during the course. Hence I should like to express my sincere gratitude to both authors for their valuable contributions. I have no doubts that the present book will be accepted most favourably by the scientific community - colleagues and students alike.

Wilhelm Schneider
Vienna University of Technology
Former Rector of CISM

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Chapter 1

GENERAL INTRODUCTION ON INSTABILITY AND TRANSITION

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1.1 Introduction

This monograph is an idiosyncratic look at the topics of research interests of the contributing authors ranging from (i) classical linear hydrodynamic instability in chapter 1; on (ii) receptivity (chapter 2); (iii) other topics of current interest on bypass transition (chapter 3) and spatio-temporal instability (chapter 4); (iv) bifurcation and nonlinear stabilization in a bluff-body flow (chapter 5) and (vi) qualitative changes in flow instability due to restricted heat transfer (chapter 6). These topics are admixtures of linear and nonlinear aspects of flow instabilities studied via analytical and computational routes. Even though in chapters 2 to 5, theoretical results are supplemented by computational results obtained by specifically developed high accuracy direct numerical simulation (DNS) techniques, in chapters 7 to 11 attention is focused on computational results obtained for combustors and combustion processes in engineering- a very specialized topic of current interest. This requires understanding complex, reacting, multiphase flows and their computations by large eddy simulation (LES). For this purpose, special attention is devoted in understanding waves in reacting flows and the role of computing errors in LES. However, over-riding all these special aspects, one needs to understand the concepts of basic fluid mechanics and fundamental principles of receptivity and instability.

Thus, the summer school at CISM in June 2008 and the present monograph grew out of a desire of the contributing authors to add those areas essential to the understanding of core materials, emphasizing those areas which have not been covered earlier. This is with a hope that this would form a timely addition to the subject across a large range of studies in flow instabilities and its dependence on heat transfer and chemical reactions.

Fluid dynamics continues to be the crucible of studies for continuum mechanics. The subject as a whole straddles fields involving many areas of engineering applications and to basic principles of instability of dynamical systems. It was not before 1687 (the year of publication of *The Principia* by Isaac Newton), the equations of motion was first written down for mechanical systems. The first proper mathematical modeling of what is now called the inviscid flow was first published by Euler in 1752. Understanding of fluid flow by empirical models during and before this period was so unsatisfactory that Robins (1746) noted that *all the theories of resistance hitherto established are extremely defective, and that it is only by experiments analogous to those here recited that this important subject can ever be completed.* It was in 1840 that the equation of fluid motion in the presence of friction –the Navier-Stokes equation - was published. The sanctity and correctness of this equation is now well-accepted in the continuum regime of fluid flow. Apart from the fact that correct governing equations are needed, one has also to ensure that correct boundary and initial conditions are used in the solution process. In this context, investigations on the validity of no-slip boundary condition have been revisited many a times. One must note that this boundary condition is a modeling approximation and has never been proven rigorously. Batchelor (1988) has therefore noted for Newtonian fluid flow that *the absence of slip at a rigid wall is now amply confirmed by direct observation and by the correctness of its many consequences under normal conditions.* There are now some exceptions to this boundary condition for low density and in micro- to nano-scale flows, that has been interestingly attributed to electrokinetic effects or flow instabilities by Tropea *et al.* (2007). Despite this, by and large we will not question the correctness of no-slip condition for continuum flows. Issues of flow instabilities and their resolution has occurred side by side with the development of the subject of fluid mechanics itself. We therefore begin with some general observations on fluid mechanics and its relation to studies of flow instabilities.

After Navier- Stokes equation has been written down in the first half of nineteenth century, few exact solutions were obtained for few fluid flows. In one such case, Stokes compared theoretical prediction with available experimental data for pipe flow and found no agreement whatsoever. Now we know that the theoretical solution of Stokes corresponded to undisturbed laminar flow, while the experimental data given to him corresponded to a turbulent flow. This problem was seized upon by Osborne Reynolds, who explained the reason for such mismatches by his famous pipe flow experiments (Reynolds, 1883). It was shown that the basic flow obtained as a legitimate solution of the governing Navier- Stokes equation is unable to maintain its stability with respect to omnipresent small disturbances in the flow. Mere mathematical existence of a solution does not always guarantee its physical realization and observation. Existence of a mathematical solution shows a possibility of a solution (that we will refer as the equilibrium solution), as it embodies satisfaction of conservation laws satisfying force, moment and energy balance . However, additionally one needs to study the stability of each and every such solutions to ascertain their observability. Reynolds demonstrated experimentally the equilibrium parabolic profile disintegrating into sinuous motion of water in the pipe that eventually led to random or turbulent flow.

In view of the above, very aptly the following is noted in Landau & Lifshitz (1959) that *the flow that occurs in nature must not only follow the equations of fluid dynamics, but also be stable.* This observation is central to many physical phenomena- where *observability* of solution is of fundamental importance. If solutions are not *observable*, then the corresponding equilibrium flows are not *stable*. Here, implication of flow *instability* is in the context of continuous deviation of the instantaneous solution from the equilibrium solution caused by growth of infinitesimally small perturbations present in the surroundings of the system. It is this sensitive dependence on, often unquantifiable, disturbance environment that makes the subject of instability very challenging. At the same time, smallness of the background disturbances allows one to study the problem of growth of these from a small perturbation approach. This greatly helps, if the governing nonlinear equations can be solved for the equilibrium solution with ease and then its stability can be studied by linearizing the governing equation for the perturbation field. In the present monograph this is amply demonstrated in chapters 1-4 in studying linear instability.

To understand better the issues affecting flow instability, certain features of the dye-experiments performed by Reynolds (1883) is worth recounting- which is perhaps the first recorded thorough experimental observations on the phenomenon of flow instability. Reynolds in his experiments, took pipes of different diameters fitted with a trumpet shaped mouth-piece or bell-mouth. The mouth-piece accelerates the flow locally, creating a favorable pressure gradient that has the propensity of attenuating background disturbances, as we will explain in chapter 2. It is also for the reason to reduce disturbances, experiments were performed by Reynolds during mid-night to avoid noise from daytime vehicular traffic. Reynolds also observed that the rapid diffusion of dye with surrounding fluid depends on the non-dimensional parameter, Va/ν with V as the center-line velocity in the pipe whose diameter is a and ν is the kinematic viscosity. This ubiquitous non-dimensional parameter is now called the Reynolds number (Re) to underscore the singular importance of this now-famous pipe flow experiment. Reynolds found that the flow can be kept orderly or laminar up to Re = 12,830. He noted that this value is very sensitive to the disturbances in the flow before it enters the tube. Thus, he noted quite prophetically that *this at once suggested the idea that the condition might be one of instability for disturbance of a certain magnitude and stable for smaller disturbances.* The relationship of instability with disturbance amplitude is a typical attribute of non-linear instability. It is now well established that pipe and plane Couette flows are linearly stable for all Reynolds numbers, when the usual linear stability analysis is performed. This therefore suggests that either non-linear and/ or different unknown linear mechanism(s) of instabilities are at play for these flows.

Later, the critical Re was further raised for pipe flows, establishing that there is perhaps no upper limit above which transition to turbulence can not be prevented. This example also suggests the importance of receptivity of flow to different types of input disturbances to the system. If there is no input to a fluid dynamical system of a particular kind that triggers instability, then the response will demonstrate the flow to be orderly, even if the fluid dynamical system is unstable to that kind of input. Thus, if the basic flow is receptive to a particular disturbance, then the equilibrium flow will not be observable in the presence of such disturbances.

Reynolds' experiment pointed out the instability as the main reason for the non-observability of basic flow, but it still did not clarify the steps following which one gets to the turbulent flow stage. Like every other subject, instability and transition also has gone through uneven progress from this pioneering experiment to its present state. We note that like many other fields, the associated scientific ideas, laws, and their discovery as narrated in textbooks on flow instabilities are mere distillation of complex multi-faceted, subtle and convoluted historical narrative. The initial impetus in any field itself could be due to missed starts, dead ends etc. and afterward it is invariably followed by mistakes, sophistries those are at times no more than self-fulfilling prophesies (aptly called the Pygmalion or Rosenthal effects in social sciences) and deceptions- that makes the whole journey a maze with errors appearing understandable, as an after-thought.

There have been significant contributions initially made by Helmholtz, Kelvin and Rayleigh (1880, 1887) using inviscid analysis. In their quest to justify their inviscid analysis, an assumption was made that viscous action due to its dissipative nature can be only stabilizing. Such was the impact of this observation that when Heisenberg (1924) submitted his dissertation solving perturbation equations including viscous terms for boundary layer (under the guidance of Sommerfeld), the examination committee could find nothing wrong in the analysis, but found it difficult to accept that viscous action can add to instability. Of course, it makes perfect sense now, when one realizes that viscous action can cause a phase delay that can lead to positive feedback and hence destabilize the flow. Similar fate awaited the researchers from the Göttingen-school led by Prandtl (1935) in developing viscous linear stability theory that explained many aspects of the early stages of transition process involving formation and growth of waves, attributed to Tollmien and Schlichting. This theory is based on the key equation developed independently by Orr (1907) and Sommerfeld (1908) and is named after them. It took the pioneering experimental effort of Dryden and his associates in establishing viscous linear stability theory by detecting the so-called Tollmien-Schlichting waves through the famous vibrating ribbon experiments of Schubauer & Skramstad (1947).

Linear stability theory results match quite well with controlled laboratory experiment for thermal and centrifugal instabilities. But, instabilities dictated by shear force do not match so well, e.g. linear stability theory applied to plane Poiseuille flow gives a critical Reynolds number of 5772, while experimentally such flows have been observed to become turbulent even at Re = 1000- as shown in Davies and White (1928). Couette and pipe flows are also found to be linearly stable for all Reynolds numbers, the former was found to suffer transition in a computational exercise at $Re = 350$ (Lundbladh & Johansson, 1991) and the latter found to be unstable in experiments for $Re \geq 1950$. Interestingly, according to Trefethen *et al.* (1993) the other example for which linear analysis fails *include to a lesser degree, Blasius boundary layer flow.* This is the flow which many cite as the success story of linear stability theory.

Flow instability of attached boundary layers has been predicted with some success and the corresponding empirical transition prediction methodologies have matured to such an extent that they are now routinely used in aircraft industry. A flat plate placed in a stream with moderately low ambient disturbance level (turbulence intensity below 0.5%), flow transition takes place at a distance x from the leading edge given by,

$$Re_{tr} = \frac{U_\infty x}{\nu} = 3.5 \times 10^5 \quad to \quad 10^6$$

The onset of instability is predicted at $Re_{cr} = 519$ (based on displacement thickness as the length scale). It is important to realize that instability and transition are not synonymous. Actual process of transition begins with the onset of instability but the completion may depend upon multiple factors those form the basis for adjunct topics like secondary, tertiary and nonlinear instabilities. Hence it is difficult to predict Re_{tr} as compared to Re_{cr}. In many external flows, the latter processes takes place over such a short streamwise distances, that the above mentioned empirical prediction methodologies neglect the distinction between the two.

It is also important to note that the linear stability theory studies a particular class of problems where the disturbances decay as one moves away in the wall-normal direction from one of the boundaries. Thus, the developed theory is mainly for disturbances that originate at the wall. The problem of destabilizing a shear layer by disturbances outside the shear layer has not received sufficient attention or adequately tackled in the past. This is also one of the major focus of this monograph, as we discuss it in chapter 2.

Despite many attempts made over more than a century, the exact route to turbulence is still far from clear. This prompted Morkovin (1991) to state: *"One hundred eight years after O. Reynolds demonstrated turbulence in a circular pipe, we still do not understand the nature of the irregular fluctuations at the wall nor the formation of larger coherent eddies convected downstream further from the wall. Neither can we describe the mechanisms of the instabilities that lead to the onset of turbulence in any given pipe nor the Reynolds number (between about 2000 and 100 000) at which it will take place. It is sobering to recall that Reynolds demonstrated this peculiar non-laminar behaviour of fluids before other physicists started on the road to relativity theory, quantum theory, nuclear energy, quarks etc"*. While some additional researches have clarified some key concepts, the situation about flow transition in a pipe remains the same.

The search for complete understanding on the origin and nature of turbulence continues- with the hope that the numerical solution of full Navier-Stokes equation without any modeling, as in DNS, may provide insight to it. Multitude of published DNS results in the literature suffers from a major drawback though, with most of them not requiring any explicit forcing of the flow via definitive and realistic input disturbance field. Most of these depend upon computational noises and /or "random noise". We wish to point out that unlike in mathematical closed-form solutions, numerical solution is always visited upon by *numerical noise* of the method and the hardware. Thus, the implicit assumption in DNS that these *numerical noise* sources will produce a *turbulent flow* and that is the same as the physical turbulent flow is a statement of hope and yet to be established rigorously. In fact, the contrary seems to be the case, as is shown in chapter 3 via DNS of bypass transition where a physical process is traced following explicit excitation. In contrast in chapters 7 to 11, computed LES solutions did not require any explicit specification or modeling of input disturbance or noise.

In studying stability of flows, it is convenient to pose the problem either as a temporal or as a spatial instability problem. While it is numerically expedient to take a temporal approach, many practical flows are known to follow spatial route. For example in lab experiments for external wall-bounded flows, it is noted that the disturbances grow in space as they travel downstream. This was established unambiguously through the experiments of Schubauer & Skramstad (1947) for flat plate boundary layer and is an excellent example of spatial instability problems. However, there are many flows where the instability grows both in space and time. These type of problems to identify whether the flow suffers temporal and/ or spatial instability arise in linear stability analysis. Flow instability studied following descriptions of two independent routes, is an artificial way of treating general instability problems.

1.2 What is Instability?

To analyze a physical problem analytically, we must obtain the governing equations that model the phenomenon adequately. Additionally, if the auxiliary equations pertaining to initial and boundary conditions are prescribed those are also well-posed, then conceptually getting the solution of the problem is straightforward. Mathematicians are justifiably always concerned with the *existence* and *uniqueness* of the solution. Yet not every solution of the equation of motion, even if it is exact, is observable in nature. This is at the core of many physical phenomena where *observability* of solution is of fundamental importance. If the solutions are not *observable*, then the corresponding basic flow is not *stable*. Here, the implication of *stability* is in the context of the solution with respect to infinitesimally small perturbations.

In studying the stability of problem with respect to ambient disturbances, it is hardly ever possible that one can incorporate all the contributing factors in a given physical scenario for posing a physical problem. Occasionally these neglected *causes* can be incorporated by *process noise* and results are made to correlate with the physical situation. This is possible when the *causes* are statistically independent and then it follows upon using Central Limit Theorem.

1.3 Temporal and Spatial Instability

Instability of an autonomous system is strictly for time-dependent systems that would display growth of disturbances in time. This may also mean that either we are studying the stability of a flow at a fixed spatial location or the full system displays identical variation in time for each and every spatial locations. In reality, many fluid flows display a growth in space, or in time, or a complex spatio-temporal growth of disturbances. For disturbances that originate from a fixed location in space the disturbance grows, as it convects downstream. Thus, the disturbance is termed unstable, if it grows unbounded as it moves downstream. This is called the convective instability. This type of instability is seen in wall bounded shear layers, exemplified by the classic vibrating ribbon experiment of Schubauer & Skramstad (1947). This experiment was performed in a very quiet facility to create Tollmien- Schlichting (TS) waves by vibrating a ribbon inside a flat plate boundary layer. This experiment was the first one to show the existence of viscous unstable waves those were predicted earlier theoretically by Heisenberg (1924), Tollmien (1931) and Schlichting (1933), but were not supported experimentally immediately. Hence, the existence of TS waves were doubted before these experimental results were known. Additionally, this experiment was also the first one that displayed the receptivity of wall-bounded shear layer to vibratory disturbances within the shear layer, while showing the inadequacy of acoustic excitation in creating TS waves.

For convectively unstable flows, disturbances are swept away from their actual origin. However, in many cases it can so happen that the disturbance can grow first in time at a fixed location, before they are convected downstream. Such growth of disturbances both in space and time are seen in many free shear layers and bluff-body flows. If we subject the equilibrium solution of such an unstable fluid dynamical system to a localized impulse, then the response field spreads both upstream and downstream of the location with respect to the local flow, where it originated while growing in amplitude. Such instabilities are termed as *absolute instabilities*. Here, an additional distinction needs to be made between convective and absolute instabilities. On application of an impulse, both the situation display disturbances in upstream and downstream directions. However, in convectively unstable system the growth of the disturbance is predominantly in one direction, while for absolutely unstable system the growth will be omni-directional. These ideas will be further clarified by understanding some basic concepts related to wave motion.

1.4 Elements of Wave Mechanics

In the previous section, we have distinguished between systems that grow either with time or with space. In reality, many flows display a complex space-time dependence for the disturbance evolution. In contrast to laminar flows, transitional and turbulent flows display broad-band spectra in wave number and circular frequencies. Thus, it facilitates to discuss such flow dynamics in spectral plane. Below, we provide a very brief introduction to wave mechanics, as applied to a model spatio-temporal dynamical system. Of special interest, is the utility of the concept of dispersion relation that relates the wave number with the circular frequencies. As we note that such dispersion relations are consequences of the governing differential equations and/ or the associated auxiliary conditions.

A wave may be viewed as a unit of the response of the system to applied input or disturbances. These responses could be in terms of physical deflections, pressure, velocity, vorticity, temperature etc., those physical properties relevant to the dynamics, showing up in general, as function of space and time. Any arbitrary function of space and time can be written in terms of Fourier-Laplace transform as given by,

$$f(x,t) = \int_{Br} \int F(\omega, k) e^{i(kx-\omega t)} d\omega dk, \tag{1.4.1}$$

for a system whose property $f(x,t)$ varies with time and single space dimension x. In this definition, ω and k are the complex circular frequency and wave number, respectively. A special point is stated here to interpret the meaning of Eqn. (1.4.1). The integrals in this equation are performed along special contours in the complex $\omega-$ and $k-$plane and are called the Bromwich contours- whose definition and usages can be further seen in Van der Pol & Bremmer (1959) and Papoulis (1962). The Bromwich contours are chosen in the strip of their convergence, where they are defined. This is discussed in section 2.6.1.2 for an elaborate explanation and many applications for boundary layer instability are shown in section 2.6. Choice of the contour in the $\omega-$plane depends on the physical principle of *causality.* In the same way, the k-plane contour has to be chosen in such a way that the poles and singularities of the response should be positioned to account for the correct directionality of the associated response field. The transforms related to time dependence are thus sought in terms of unilateral Laplace-Fourier transform and the transforms associated with space dependence are defined via bi-lateral Laplace transform. Equation (1.4.1) is the inverse transform to obtain the function in the physical space based on the value of the direct transform $F(\omega, k)$, obtained in the spectral plane. The direct transform can be obtained in turn from the following,

$$F(\omega, k) = \int_{-\infty}^{\infty} \int_{0}^{\infty} f(x,t) e^{-i(kx-\omega t)} dx dt \tag{1.4.2}$$

In writing the above transform, it is implied that the input to the system is applied at $t = 0$, and thus, the time integral starts at that time, essentially following the *causality* principle. However no such restriction needs to be applied for space dependence of the function.

The definition integrals of Eqns. (1.4.1) and (1.4.2), tell us that any arbitrary space-time dependent functions can be thought of as an ensemble of large numbers of waves with different combinations of wave numbers and circular frequencies. This assembly could be a result of countably infinite numbers of waves or it could represent a continuous spectrum. There is a definitive relationship between the wave numbers and the circular frequencies, as identified before, as the dispersion relation.

Let us explain the implication of dispersion relation through a simple example of one-dimensional wave propagation whose governing equation is given by,

$$\frac{\partial f}{\partial t} + c\frac{\partial f}{\partial x} = 0 \tag{1.4.3}$$

In this equation c is the phase speed of the wave. We know that the exact solution of Eqn. (1.4.3) shows the initial solution to travel to the right at the phase speed (c). If we use the representation given by Eqn. (1.4.1), in Eqn. (1.4.3) we then get,

$$\int_{Br}\int i(kc-\omega)Fe^{-i(kx-\omega t)}d\omega dk = 0 \tag{1.4.4}$$

Since the integral is true along the Bromwich contours and is equal to zero, hence a non-trivial and unique solution would require the following relation to hold,

$$\omega = kc$$

This is the desired relationship between the circular frequency and the wave number and is the *dispersion relation.* For a general problem governed by other forms of equation, the *dispersion relation* will be of the kind,

$$\omega = \omega(k) \tag{1.4.5}$$

Thus, the dispersion relation for Eqn. (1.4.3), is the statement of governing equation in the spectral plane and tells us that the scale of space variation and the scale of time variation are not independent and they are related. For many other problems, the dispersion relation will be consequence of boundary conditions, as is often derived for water waves developing for an equilibrium solution given by the Laplace's equation. Equation (1.4.5) implies that each frequency component will travel in space with the corresponding phase speed. Any system that has relationship showing that the phase corresponding to each wave number travels by the same speed is called a *non-dispersive system*. In contrast, a *dispersive system* is one for which different frequency components travel with different speed, such that any compact disturbance at $t = 0$ will be found dispersed at a later time. One can generalize the above for a system that displays broad-band dynamics.

Once again for such a general system, the governing differential equation(s) and the boundary condition(s) will determine the *dispersion relation*. Once we have this relation, it is possible to talk about the general properties of the response field (Lighthill(1978)). Let us consider two neighboring wave numbers, k_1 and k_2 of the response field. For the physical problem, response field depends continuously on the forcing and auxiliary conditions. Hence the corresponding circular frequencies, ω_1 and ω_2, as obtained from the *dispersion relation* will also be two closely spaced neighboring points in the frequency plane. We denote the corresponding response components by,

$$f_1 = a\cos(k_1x - \omega_1 t) \tag{1.4.6a}$$

$$f_2 = a\cos(k_2x - \omega_2 t) \tag{1.4.6b}$$

We consider here only the real part of the response field. A more generalized response based on the expression (1.4.1) will have also an imaginary component signifying the phase shift of the output with respect to applied input. In the above equations, both the components have been assumed to have same amplitude, the difference being negligible- as we are considering two components that are separated by an infinitesimal amount in the spectral plane. The total contribution coming from these two components is then given by,

$$f = f_1+f_2 = [2a\cos\frac{1}{2}\{(k_1-k_2)x-(\omega_1-\omega_2)t\}]\cos\frac{1}{2}\{(k_1+k_2)x-(\omega_1+\omega_2)t\} \tag{1.4.7}$$

Total effect of these two components shows up as a combination of two factors- one on a slow scale given by the first quantity inside the square bracket and the second quantity that varies on the original scale given by the second cosine function. As the two components are infinitesimally apart in the spectral plane, the slowly varying part can be viewed as the amplitude of the total effect. As energy of a wave system is proportional to the square of the amplitude of the wave, one can view the energy of the system to vary following the phase variation of the amplitude. Therefore the speed of propagation of energy is given by the rate at which the phase of the amplitude changes and is given by,

$$V_g = \frac{(\omega_2 - \omega_1)}{(k_2 - k_1)}$$

This is the group velocity of the simple system considered here, where the spatial variation is along a single direction and the group velocity direction is given along this direction. For the one-dimensional system in the limit, when the wave numbers of the two components are only dk apart in $k-$space, then one can consider an equivalent response field centered around this cluster and a general expression for the group velocity is given by,

$$V_g = \frac{d\omega}{dk} \tag{1.4.8}$$

The group velocity for multi-dimensional problem is a vector, decided by the variation of circular frequency with the wave number vector. It is only the real part of Eqn. (1.4.8) that is termed the group velocity - as discussed in Whitham (1978). Thus, this is the velocity at which the energy of a group of waves travel, centered about the middle of the wave number group. It is noted that the one-dimensional wave given by Eqn. (1.4.3), is non-dispersive with $V_g = c$.

Regarding the classification of instabilities into convective and absolute instability, one can now see the difference clearer in terms of the group velocity. For absolute instability the group velocity is found to be zero, so that the disturbances do not get swept away, as in convective instability and continue to grow in the place of their origin. However, in many flow systems, these two aspects can remain simultaneously.

1.5 Some Instability Mechanisms

Here two simple cases of instabilities are considered to emphasize the concepts described above. We begin by distinguishing the difference between static and dynamic instability by considering the stability of atmosphere as an example.

When a parcel of air in the atmosphere is moved rapidly from an equilibrium condition and its tendency to come back to its undisturbed position is noted, then we term the atmosphere as statically stable. The movement of the packet is considered as impulsive, to preclude any heat transfer from the parcel to the ambience. This tendency of static stability- when exists, is due to the buoyancy force caused by the density differential due to temperature variation with height and such body force acts upon the displaced air-parcel.

In static stability studies, we do not look for detailed timed-dependent motion of the parcel following the displacement (as the associated accelerations are considered negligible).

1.5.1 Dynamic Stability of Still Atmosphere

For the dynamic stability study, we consider once again a parcel of air to be at equilibrium, at a height z and is displaced to a height $(z+\xi)$ to follow its detailed time history of motion. We note that the motion is caused again by the buoyancy force, caused by the temperature variation of the ambient air given by $T = T(z)$. Here, the vertical displacement of the air-parcel is ξ and the dynamics follows the force balance equation,

$$\rho' \ddot{\xi} = g(\rho - \rho')_{z+\xi} \tag{1.5.1}$$

where $\ddot{\xi}$ is the instantaneous acceleration experienced by the parcel of air. Density of the displaced parcel is considered to be given by ρ', while the ambient fluid has the density ρ, so that the right hand side of the above equation represents the buoyancy force. Equilibrium thermodynamics tells us that for a simple compressible substance with only one mode of work, any state property can be represented by any two other properties and let us consider them to be the pressure (p) and the entropy (s). Thus, using a Taylor series we can relate the density of the ambient air at the two heights as

$$\rho(z+\xi) = \rho(z) + \left(\frac{\partial \rho}{\partial p}\right)_s [p(z+\xi) - p(z)] + \left(\frac{\partial \rho}{\partial s}\right)_p [s(z+\xi) - s(z)] + ... \tag{1.5.2}$$

Once again, we will assume that the displacement process of the air-parcel is isentropic (there are no viscous or heat losses associated with the rapid movement of the parcel) and thus for the air-parcel,

$$\rho'(z+\xi) = \rho'(z) + \left(\frac{\partial \rho}{\partial p}\right)_s [p(z+\xi) - p(z)] \tag{1.5.3}$$

In Eqns. (1.5.2) and (1.5.3), mechanical equilibrium ensures same δp and $\rho(z) = \rho'(z)$. Thus, the density differential causing the buoyancy is given by,

$$(\rho - \rho')_{z+\xi} = \left(\frac{\partial \rho}{\partial s}\right)_p [s(z+\xi) - s(z)] = \left(\frac{\partial \rho}{\partial s}\right)_p \frac{ds}{dz}\xi \tag{1.5.4}$$

We can also relate the density of the air-parcel at the two heights as,

$$\rho'(z+\xi) = \rho(z) + \left(\frac{\partial \rho}{\partial p}\right)_s \frac{dp}{dz}\xi = \rho(z) + \frac{1}{c^2}\frac{dp}{dz}\xi$$

where c is the speed of sound. Equation (1.5.1) can also be written in terms of the specific volume ($v = \frac{1}{\rho}$), using Eqn. (1.5.4) as,

$$\ddot{\xi} = \frac{g}{\rho'}(\rho - \rho')_{z+\xi} = -\left[\frac{g}{v}\left(\frac{\partial v}{\partial s}\right)_p \frac{ds}{dz}\xi\right] / \left[1 + \frac{v\xi}{c^2}\frac{dp}{dz}\right]$$

From the mechanical equilibrium, $\frac{dp}{dz} = -\rho g$, above can be further simplified to

$$\ddot{\xi} = -\left[\frac{g}{v}\left(\frac{\partial v}{\partial s}\right)_p \frac{ds}{dz}\xi\right] / \left[1 - \frac{g\xi}{c^2}\right]$$

We can further simplify by using the thermodynamic relations: $\left(\frac{\partial v}{\partial s}\right)_p = \left(\frac{\partial T}{\partial s}\right)_p \left(\frac{\partial v}{\partial T}\right)_p$ and $\frac{ds}{dz} = \left(\frac{\partial s}{\partial T}\right)_p \frac{dT}{dz} + \left(\frac{\partial s}{\partial p}\right)_T \frac{dp}{dz}$, noting that $\left(\frac{\partial s}{\partial T}\right)_p = \left(\frac{\partial s}{\partial h}\right)_p \left(\frac{\partial h}{\partial T}\right)_p = \frac{C_p}{T}$. From the Maxwell's relation, $\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p$, we obtain $\frac{ds}{dz} = \frac{C_p}{T}\frac{dT}{dz} + \rho g \left(\frac{\partial v}{\partial T}\right)_p$.

All these simplifications lead to,

$$\ddot{\xi} = -\frac{g\xi}{v}\left(\frac{\partial v}{\partial T}\right)_p \frac{T}{C_p}\left[\frac{C_p}{T}\frac{dT}{dz} + \frac{g}{v}\left(\frac{\partial v}{\partial T}\right)_p\right] / \left[1 - \frac{g\xi}{c^2}\right]$$

If we consider air as a perfect gas ($p = \rho RT$), then $\left(\frac{\partial v}{\partial T}\right)_p = v/T$ and the above further simplifies to

$$\ddot{\xi} = -\frac{g}{T}\left(\frac{dT}{dz} + \frac{g}{C_p}\right)\xi / \left[1 - \frac{g\xi}{c^2}\right]$$

If we further consider the speed of sound (c) to be very large, then the above equation can be further approximated to

$$\ddot{\xi} + N^2 \xi = 0 \tag{1.5.5}$$

where $N^2 = \frac{g}{v}(\frac{\partial \nu}{\partial s})_p \frac{ds}{dz}$. We can consider the following possibilities:

Case-1: If $N^2 > 0$ then the dynamics of the displacement will be purely oscillatory, implying neutral stability of the static atmosphere.

Case-2: If $N^2 < 0$, then the vertical displacement will vary as,

$$\xi(t) = Ae^{|N|t} + Be^{-|N|t}$$

where the first component clearly indicates instability. N is called the *Brunt-väisälä* or buoyancy frequency. As given above, following Thompson (1972), we can obtain this frequency with air treated as an ideal gas by,

$$N^2 = \frac{g}{T}\left[\frac{dT}{dz} + \frac{g}{C_p}\right] \tag{1.5.6}$$

For dry air, $\frac{g}{C_p} = -0.01$ K/meter and hence for stability of dry atmosphere the temperature distribution has to be so that $\frac{dT}{dz} > -0.01$ K/meter. Thus $\frac{dT}{dz} = 0.01$ represents the border line of instability and the numerical value on the right hand side within the square bracket is termed as the dry adiabatic lapse rate, because this ensures $\frac{ds}{dz} = 0$.

1.5.2 Kelvin - Helmholtz Instability

This arises when two layers of fluids (may not be of same species or density) are in relative motion. Thus, this is an interfacial instability and the resultant flow features due to imposed disturbance will be much more complicated due to relative motion. Physical relevance of this problem was seized upon by Helmholtz (1868) who observed that the interface as a surface of separation tears the flow *asunder*. Sometime later Kelvin (1871) posed this problem as one of instability and solved it. We follow this latter approach here. The basic equilibrium flow is assumed to be inviscid and incompressible - as two parallel streams having distinct density and velocity - flowing one over the another, as depicted in figure below.

Before any perturbation is applied, the interface is located at $z = 0$ and subsequent displacement of this interface is expressed parametrically as,

$$z_s = \hat{\eta}(x, y, t) = \epsilon\eta(x, y, t) \tag{1.5.7}$$

where ϵ is a small parameter, defined to perform a linearized perturbation analysis. One can view the interface itself as a shear layer of vanishing thick-

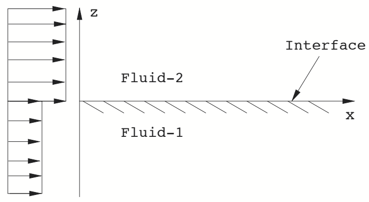

Figure 1.1 Kelvin-Helmholtz instability at the interface of two flowing fluids.

ness. For the considered inviscid irrotational flows, the velocity potentials in the two domains are given by,

$$\tilde{\phi}_j(x, y, z, t) = U_j x + \epsilon\phi_j(x, y, z, t) \tag{1.5.8}$$

The governing equations in either of the flow-domains are given by,

$$\nabla^2\tilde{\phi}_j = 0 \tag{1.5.9}$$

And the potential must satisfy the following far-stream boundary conditions given by,

$$\phi_j s \quad are \quad bounded \quad as \quad z \rightarrow \pm\infty \tag{1.5.10}$$

The other set of boundary condition is applied at the interface, which is the no-fluid through the interface condition i.e.

$$\frac{\partial\hat{\eta}}{\partial t} - \frac{\partial\tilde{\phi}_j}{\partial z} = -\frac{\partial\hat{\eta}}{\partial x}\frac{\partial\tilde{\phi}_j}{\partial x} - \frac{\partial\hat{\eta}}{\partial y}\frac{\partial\tilde{\phi}_j}{\partial y} \tag{1.5.11}$$

In addition, in the absence of surface tension, pressure must be continuous across the interface. Upon linearization, the interface boundary condition (1.5.11) simplifies to,

$$\frac{\partial\eta}{\partial t} + U_j\frac{\partial\eta}{\partial x} - \frac{\partial\phi_j}{\partial z} = 0 \quad for \quad j = 1, \quad 2 \tag{1.5.12}$$

where $\tilde{\phi}_j$ and ϕ_j are as related in Eqn. (1.5.8). Defining the pressure on the either flow-domain by unsteady Bernoulli's equation, one can write

$$p_j = C_j - \rho_j\left\{\frac{\partial\tilde{\phi}_j}{\partial t} + \frac{1}{2}(\nabla\tilde{\phi}_j)^2 + g\hat{\eta}\right\} \tag{1.5.13}$$

Simplifying and retaining up to $0(\epsilon)$ terms, we get the following conditions

$$0(1)\ \ condition: C_1 - \frac{1}{2}\rho_1 U_1^2 = C_2 - \frac{1}{2}\rho_2 U_2^2 \tag{1.5.14a}$$

$$0(\epsilon)\ \ condition: \rho_1\left\{\frac{\partial\phi_1}{\partial t} + U_1\frac{\partial\phi_1}{\partial x} + g\eta\right\} = \rho_2\left\{\frac{\partial\phi_2}{\partial t} + U_2\frac{\partial\phi_2}{\partial x} + g\eta\right\} \tag{1.5.14b}$$

One can consider a very general interface displacement given in terms of bilateral Laplace transform as,

$$\eta(x, y, t) = \int\int F(\alpha, \beta, t)e^{i(\alpha x+\beta y)}d\alpha d\beta \tag{1.5.15}$$

Correspondingly, the perturbation velocity potential is expressed as,

$$\phi_j(x, y, z, t) = \int\int Z_j(\alpha, \beta, z, t)e^{i(\alpha x+\beta y)}d\alpha d\beta \tag{1.5.16}$$

Writing $k^2 = \alpha^2 + \beta^2$ and using Eqn. (1.5.16) in (1.5.9), one gets the solution that satisfies the far-stream boundary conditions (1.5.10) as,

$$Z_j = f_j(\alpha, \beta, t)e^{\pm kz} for \quad j = 1 \ \ and \quad 2 \tag{1.5.17}$$

Using Eqn. (1.5.15) in the interface boundary condition (1.5.12) one gets,

$$\dot{F} + i\alpha U_1 F - kf_1 = \dot{F} + i\alpha U_2 F + kf_2 = 0 \tag{1.5.18}$$

where the dots denote differentiation with respect to time. If we denote the density ratio $\rho = \rho_2/\rho_1$, then the linearized pressure continuity condition (1.5.14b) gives,

$$\frac{\partial\phi_1}{\partial t} - \rho\frac{\partial\phi_2}{\partial t} + U_1\frac{\partial\phi_1}{\partial x} - \rho U_2\frac{\partial\phi_2}{\partial x} + (1-\rho)g\eta = 0 \tag{1.5.19}$$

Using (1.5.15) and (1.5.16) in the above equation, one gets

$$\dot{f}_1 - \rho\dot{f}_2 + i\alpha U_1 f_1 - i\alpha\rho U_2 f_2 + (1-\rho)gF = 0 \tag{1.5.20}$$

Eliminating f_1 and f_2 from Eqn. (1.5.20) using Eqn. (1.5.18), one gets after simplification,

$$(1+\rho)\ddot{F} + 2i\alpha(U_1 + \rho U_2)\dot{F} - \{\alpha^2(U_1^2 + \rho U_2^2) - (1-\rho)gk\}F = 0 \tag{1.5.21}$$

This ordinary differential equation for the time variation of the interface displacement F can be understood better in terms of its Fourier transform defined by,

$$F(.,t) = \int \hat{F}(.,\omega)e^{i\omega t}d\omega \tag{1.5.22}$$

One obtains the following dispersion relation by substitution of (1.5.22) in (1.5.21) as,

$$-\omega^2(1+\rho) - 2\alpha\omega(U_1+\rho U_2) + (1-\rho)gk - \alpha^2(U_1^2+\rho U_2^2) = 0 \tag{1.5.23}$$

This provides the characteristic exponents in (1.5.22) as,

$$\omega_{1,2} = -\frac{\alpha(U_1+\rho U_2)}{(1+\rho)} \mp \frac{\sqrt{gk(1-\rho^2) - \alpha^2\rho(U_1-U_2)^2}}{(1+\rho)} \tag{1.5.24}$$

Based on this dispersion relation, the following sub-cases can be considered:

CASE 1: When the interface is disturbed in the spanwise direction only i.e. $\alpha = 0$ and then

$$\omega_{1,2} = \mp\sqrt{g\beta\frac{(1-\rho)}{(1+\rho)}} \tag{1.5.25}$$

Thus, the streaming velocities U_1 and U_2 do not affect the response of the system. If in addition, $\rho > 1$, i.e. a heavier liquid is over a lighter liquid, then the buoyancy force causes temporal instability (if β is considered real) - as is the case for Rayleigh-Taylor instability (see Chandrasekhar (1960)).

CASE 2: For a general interface perturbation if $gk(1-\rho^2) - \alpha^2\rho(U_1 - U_2)^2 < 0$, then the interface displacement will grow in time. This condition can be alternately stated as a condition for instability as: $(U_1 - U_2)^2 > \frac{gk}{\alpha^2}\left(\frac{1-\rho^2}{\rho}\right)$.

Thus, for a given shear at the interface given by, $(U_1 - U_2)$ and for a given oblique disturbance propagation direction at the interface indicated by the wave number vector k, instability would occur for all wave numbers k^*, given by

$$k^* > \left(\frac{k^*}{\alpha}\right)^2 \frac{g}{(U_1-U_2)^2}\left(\frac{\rho_1}{\rho_2} - \frac{\rho_2}{\rho_1}\right)$$

Note that the wave number vector makes an angle γ with the x-axis, such that $cos\gamma = \frac{\alpha}{k^*}$ and the above condition can be conveniently written as,

$$k^* > \frac{g}{(U_1 - U_2)^2 cos^2\gamma}\left(\frac{\rho_1}{\rho_2} - \frac{\rho_2}{\rho_1}\right) \tag{1.5.26}$$

The lowest value of wave number ($k^* = k_{min}$) would occur for two-dimensional disturbances i.e. when $cos\gamma = 1$ and this is given by,

$$k^*_{min} = \frac{g}{(U_1 - U_2)^2}\left(\frac{\rho_1}{\rho_2} - \frac{\rho_2}{\rho_1}\right) \tag{1.5.27}$$

CASE 3: Consider the case of shear only of same fluid in both the domain i.e. $\rho = 1$. The characteristic exponents then simplify to,

$$\omega_{1,2} = -\alpha\frac{U_1 + U_2}{2} \mp \frac{i\alpha}{2}(U_1 - U_2) \tag{1.5.28}$$

Presence of the imaginary part with negative sign implies temporal instability for all wave lengths. Also, to be noted that since the group velocity and phase speed in y-direction is identically zero, therefore the Kelvin-Helmholtz instability for pure shear always will lead to two-dimensional instability.

Chapter 2

INSTABILITY AND TRANSITION IN FLUID MECHANICS

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2.1 Introduction

The basic aim of the material in this chapter is to acquaint readers with the state-of-art in the study of receptivity and relate the same with instability and transition of fluid flow. This subject area remains the pacing item in understanding many natural phenomena, as well as, in the analysis and design of many engineering systems. For example, this is pursued in civil aviation to design newer lifting surfaces with drag reduced by passive means. In this context, keeping the flow laminar (stable) over a wing, to as large an extent possible is the primary goal. With reduced drag, the aircraft speed and range can be increased for the same power consumed or have a less powerful engine for the same endurance and range of the flight envelope.

It is well known that for a zero pressure gradient flat plate boundary layer, the skin friction for laminar flow is given by, $C_f = \frac{1.328}{\sqrt{Re}}$ that at a Reynolds number of 10^7 works out as 0.00043 (See Schlichting (1979)) and the same profile drag increases to 0.0035 for the equivalent turbulent flow (Van Driest (1951)). This is the rationale for trying to keep a flow laminar so that one can obtain an order of magnitude drag reduction in the relevant portion of the aerodynamic surface. Such drag reductions are also realizable for airfoil- the quintessential lifting surface element of all aircraft wings. According to Viken (1983), flow past an airfoil at moderate Reynolds number, that is fully turbulent without any separation, displays a profile drag coefficient of nearly 0.0085 and that can be reduced to 0.0010 if the flow over the airfoil is maintained fully laminar. Even a modest viscous drag reduction via transition delay can provide large benefits, if large numbers of aircrafts are involved- as is the case for civil air transport industry.

Thus, transition delay for the flow over aircraft wings takes on added importance when it is realized that by resorting to transition delay techniques on wings alone, about 10 to 12% drag reduction is feasible on a modern transport aircraft. It is also now well established that transition to turbulence relates to the understanding of vorticity distribution in the shear layer and their reorganization, in response to forcing by environmental disturbances. While the flows, either in its spatio-temporal orderly form (in laminar flows) or in its chaotic form, are governed by the same generalized Navier- Stokes equation (Goldstein (1938)). The late stages of transition processes or the fully turbulent flows are not amenable to easy understanding due to the intractable nonlinearity of the Navier-Stokes equation (Morkovin (1991)). However, the onset of the transition process (also known as the receptivity stage - see Morkovin (1958, 1978, 1990)) is bedeviled by our inability to catalogue and quantify the background omnipresent disturbances. At this point in time, significant understanding of the receptivity and the linear stage of transition have been made- that allows attempts made to design aircrafts with reduced drag via transition delay. In Fig. 2.1, we reproduce the system portrait given in Morkovin (1991) for the route to wall bounded turbulent flows. As of today, the primary approaches in transition delay relates to suppressing disturbance growth during the receptivity and primary instability stages. Late stages of transition and turbulent states of flows can be controlled by active means. This aspect is still an active area of research and is currently not used in any operational transport aircrafts. Flow control is gaining in importance in recent times.

Primarily laminar boundary layers are sustained by either small amount of suction (Pfenninger (1947)) or by favourable pressure gradients. Stabilizing by suction though very efficient [critical Reynolds number increases by 90 times to $Re_{cr} = 46,000$ for asymptotic suction, as compared to the no-suction case for Blasius boundary layer for which $Re_{cr} = 520$ - see Hughes & Reid (1965) and White (1991)] - is not practiced due to operational difficulties. For example, one must have the provision of porous surface together with the necessary suction system- complexity and maintenance of such system makes this technology unattractive, at present.

In contrast, stabilization of boundary layer by contouring the airfoil surface to achieve favourable pressure gradient as a passive way is found to be practical and attractive. The resultant section is known as the Natural Laminar Flow (NLF) airfoil and this is an area which has been under renewed investigation over the last three decades. Early efforts of designing airfoils to delay transition are as given in Jacobs (1939), Tani (1952), Abbott & Doenhoff (1959). Typical examples of such airfoils designed are given by the six digit NACA series airfoils. These aerofoils were successful at low Reynolds numbers- as evidenced by their continued usage in gliders. This is due to the fact that the early NLF airfoils exhibited low drag only for a narrow range of C_l's (designed considering cruise condition only). However, such a section does not perform optimally in other sector of flight envelope. This highlights that a practical NLF airfoil must only have low cruise drag, but also must provide high lift characteristics- very essential during landing, take off and climb. Unfortunately, to maximize desired performance in terms of low drag degrades high lift performance and vice versa.

The above discussion pertaining to drag reduction of external flow over an aircraft wing is equally relevant to the power requirements for flow of water or oil in a pipeline. If the flow can be retained laminar and flow instability prevented and/or delayed, then there is a direct benefit in terms of energy efficiency. This is a justified motivation to discuss about flow instability, without the understanding of which it is not possible to design any system whose performance is dictated by fluid flow. Here we provide a general introduction to topics of receptivity and flow instability relevant to many engineering systems.

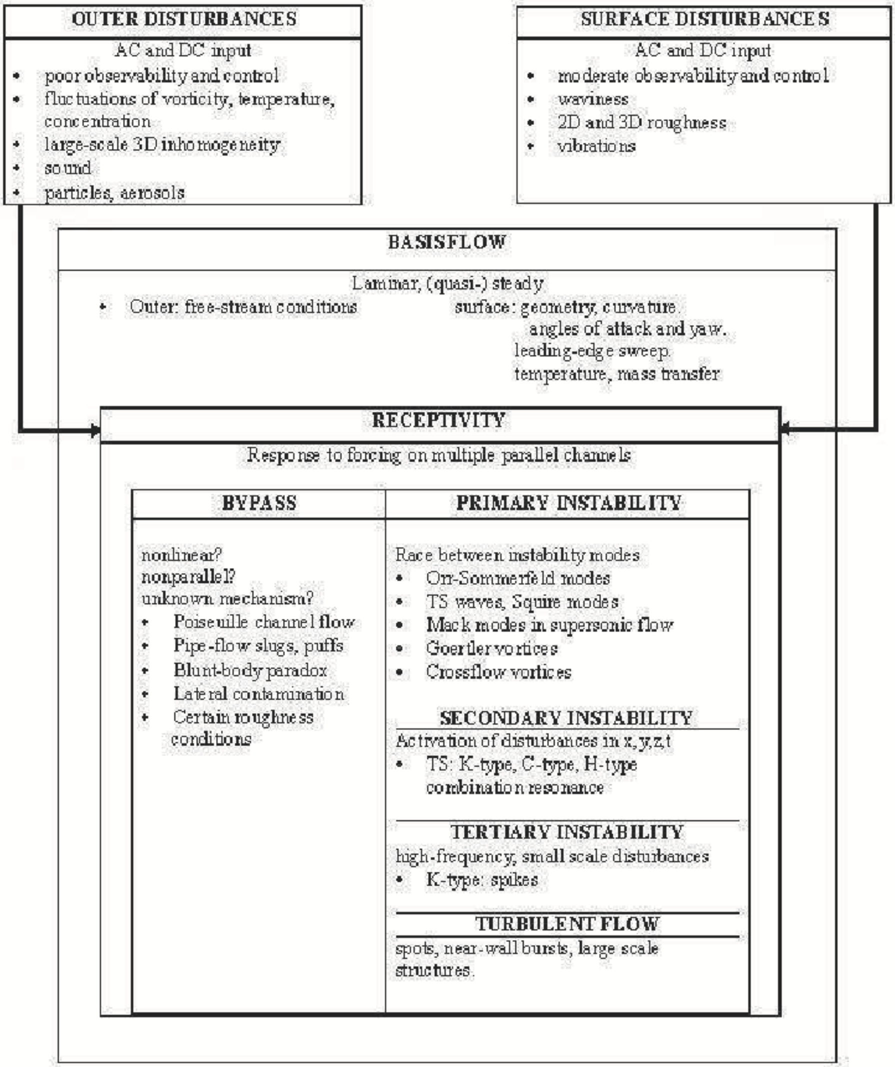

Figure 2.1 System portrait for road to turbulence (Morkovin, 1991)